UNITED STATES DEPARTMENT OF AGRICULTURE  
Rural Utilities Service  

RUS Bulletin 1724E-301  

SUBJECT: Guide for the Evaluation of Large Power Transformer Losses  

TO: All Electric Borrowers, Consulting Engineers, and Rural Utilities Service Staff  

EFFECTIVE DATE: Date of Approval  

OFFICE OF PRIMARY INTEREST: Transmission Branch, Electric Staff Division.  


AVAILABILITY: This bulletin can be accessed on the Internet at:  

http://www.usda.gov/rus/electric/bulletins.htm  

PURPOSE: This bulletin provides guidelines for evaluating transformer losses and including such losses in bid evaluations.  

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3/4/09 Date
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ABBREVIATIONS

A  Base cost before inflation
A’ Cost adjusted for inflation
FRC Fixed charge rate for capital investment expressed as a decimal in dollars per dollar of investment
EC Cost of energy in dollars per kilowatt-hour
EC’ Energy charge adjusted for inflation
G Peak ratio which is the ratio of peak load to full rated load
kVA kilovoltampere
kW kilowatt
kWh kilowatt-hour
K Peak responsibility factor which is the ratio of transformer load at the time of the system peak to the transformer peak load
LFA Loss factor for the auxiliary equipment
LFT Transformer loss factor which is the ratio of average transformer losses to peak transformer losses
LS Equivalent level losses; value that results in the same total losses as the yearly increasing load
MVA Transformer load, megavoltamperes
<table>
<thead>
<tr>
<th>OA</th>
<th>Self-cooled rating for oil-filled power transformers</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>The rate of the increase in costs per kWh associated with power generation and transmission expressed as a decimal</td>
</tr>
<tr>
<td>PW</td>
<td>Present worth</td>
</tr>
<tr>
<td>SI</td>
<td>The system capital investment in dollars per kilowatt required to supply the power losses of the transformer</td>
</tr>
<tr>
<td>TAL</td>
<td>Losses due to transformer auxiliary equipment in kilowatts</td>
</tr>
<tr>
<td>TLL</td>
<td>Transformer’s guaranteed load losses in kilowatts</td>
</tr>
<tr>
<td>TNLL</td>
<td>Transformer’s guaranteed no-load losses in kilowatts</td>
</tr>
<tr>
<td>Y₁</td>
<td>The ratio of load to capacity when transformer is installed</td>
</tr>
<tr>
<td>Y₂</td>
<td>The ratio of load to capacity when transformer is removed</td>
</tr>
<tr>
<td>r'</td>
<td>Equivalent inflation rate</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Losses and purchase price should be considered when deciding which transformer to purchase. The purpose of this bulletin is to present a uniform approach that can be used to determine the dollar value of these losses over the life of the transformer. Below is typical wording of a transformer loss evaluation clause for insertion into bidding documents that specifies how losses will be evaluated.

| “Load, no-load and auxiliary losses at 50 MVA for the 30/40/50 MVA transformer will be evaluated as follows:” |
| No-Load Losses | Load Losses | Auxiliary Losses |
| $/kW 2450 | $/kW 1304 | $/kW 756 |

The cost of losses for each transformer will be calculated by multiplying the appropriate dollars/kW values above by the guaranteed load losses at 55 °C rating and no-load losses at 100% voltages. This cost will be added to the bid price for evaluation.”

1.1 Example: Using the loss evaluation factors given above, determine which manufacturer’s transformer has the lowest evaluated cost including losses.

<table>
<thead>
<tr>
<th>161/34.5 kV, 30/40/50 MVA Transformer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer A’s Transformer</td>
</tr>
<tr>
<td>Bid price</td>
</tr>
<tr>
<td>No -load losses</td>
</tr>
<tr>
<td>Load losses at 50 MVA, 55°C temperature rise</td>
</tr>
<tr>
<td>Auxiliary losses at 50 MVA 55°C temperature rise</td>
</tr>
</tbody>
</table>
### 1.2 Solution

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid Price</td>
<td>= $424,500</td>
<td>= $436,000</td>
</tr>
<tr>
<td>Total cost of no-load losses</td>
<td>59 kW (2450 $/kW)  = $144,550</td>
<td>53 kW (2450 $/kW)  = $129,850</td>
</tr>
<tr>
<td>Total cost of load losses</td>
<td>224 kW (1304 $/kW) = $292,096</td>
<td>218 kW (1304 $/kW) = $284,272</td>
</tr>
<tr>
<td>Total cost of auxiliary losses</td>
<td>2.0 kW (756 $/kW) = $1,512</td>
<td>2.5 kW (756 $/kW) = $1,890</td>
</tr>
<tr>
<td>TOTAL COST</td>
<td>= $862,658</td>
<td>= $852,012</td>
</tr>
</tbody>
</table>

Although the transformer from Manufacturer A has the lowest bid price, the transformer from Manufacturer B has the lowest evaluated total cost.

In addition to giving loss evaluation values, the bid documents should also have penalty values that the manufacturer is to be charged for every kilowatt by which the actual tested transformer losses exceed the guaranteed losses upon which the bids are evaluated. It is important to have such penalty values in order to give an incentive to the manufacturers to provide the most accurate guaranteed loss values possible. The penalty values should be expressed in the same dollars per kW manner as the bid evaluation values but should be somewhat higher. An increment of approximately 20 percent is recommended.

### 2. FORMULAE

The three different types of transformer losses that should be evaluated separately are:

- **a.** Load losses (sometimes called copper or coil losses);
- **b.** No-load losses (sometimes called core or iron losses); and
- **c.** Auxiliary losses (electric fan losses, other such equipment losses).

Load losses are primarily from the $I^2R$ losses in the transformer windings and eddy current losses. If a value of load losses is not directly given, load losses can be determined by subtracting no-load losses from total losses. No-load losses consist of the hysteresis and the eddy current losses in the iron core of the transformer and the $I^2R$ losses in the windings due to the excitation current. Auxiliary losses consist of the power necessary to drive the auxiliary cooling pumps and fans.

---

*If the total losses at full load are 100 kW and the no-load losses are 10 kW, then the load (or copper or coil) losses are 90 kW.*
The formulae below yield the total costs of the losses that should be added to the purchase price of the transformer as shown in the Example 1.1:

\[
\text{(Cost of no-load losses in dollars)} = \left[ SI + \frac{8760 \cdot (EC)}{FCR} \right] \cdot TNLL \quad (Eq. 1)
\]

\[
\text{(Cost of load losses in dollars)} = \left[ SI(K^2)(G) + \frac{8760 \cdot (EC)(LFT)(G)}{FCR} \right] \cdot TLL \quad (Eq. 2)
\]

\[
\text{(Cost of auxiliary losses in dollars)} = \left[ SI(K^2) + \frac{8760 \cdot (EC)(LFA)}{FCR} \right] \cdot TAL \quad (Eq. 3)
\]

where:

\( G \) = peak ratio

\( K \) = peak responsibility factor

\( SI \) = the system capital investment in dollars per kilowatt required to supply the power losses of the transformer;

\( 8760 \) = the number of hours in a year;

\( EC \) = the cost of energy in dollars per kilowatt-hour;

\( FCR \) = fixed charge rate for capital investment expressed as a decimal in dollars per dollar of investment;

\( LFA \) = the loss factor for auxiliary equipment;

\( LFT \) = the transformer loss factor which is the ratio of average transformer losses to peak transformer losses;

\( TNLL \) = the transformer’s guaranteed no-load losses in kilowatts;

\( TLL \) = the transformer’s guaranteed load losses in kilowatts;

\( TAL \) = the losses due to transformer auxiliary equipment in kilowatts

A detailed discussion of the factors in Equations 1 through 3 follows in Section 3.
3. VALUES FOR FORMULAE

3.1 SI: The System Investment (SI) charge is the cost of generation and transmission facilities per kilowatt necessary to supply the additional demand resulting from the transformer losses at the system peak. Since a transformer located directly at a generating station does not require an investment in transmission facilities, the SI value used to evaluate the losses in the generating station transformer should be less than the SI of a transformer to be located at the receiving end of a transmission line.

One method for determining the SI value involves adding the construction cost (dollars per kilowatt) of a recently completed or soon to be completed generating station to the cost of the transmission facilities (dollars per kilowatt) required to connect the transformer to the plant. If power is purchased rather than self-generated, the SI value can be determined by dividing the demand charge in dollars per kW per year by the fixed charge rate (FCR). Since there is more than one method of evaluating the SI value, the method that is judged to yield the most realistic results should be used.

3.2 FCR: The fixed charge rate (FCR) represents the yearly income necessary to pay for a capital investment. FCR is expressed as a percentage of capital investment. The rate covers all costs that are fixed and do not vary with the amount of energy produced. The rate includes interest, depreciation, taxes, insurance, and those operations and maintenance expenses that do not depend on system kilowatt-hours sold.

The interest rate used should be the same as the interest rate of the loan acquired to purchase the transformers. If loan funds are not used, a blended rate of the interest earned on deposited funds should be used.

The practice of including some operations and maintenance expenses in the fixed charge rate is a matter of judgment. Some typical values for the components of the carrying charge rate are as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>7.50%</td>
</tr>
<tr>
<td>Depreciation</td>
<td>2.75%</td>
</tr>
<tr>
<td>Insurance</td>
<td>0.60%</td>
</tr>
<tr>
<td>Taxes</td>
<td>1.00%</td>
</tr>
<tr>
<td>Operations and Maintenance</td>
<td>2.76%</td>
</tr>
<tr>
<td>Carrying Charge Rate</td>
<td>14.61%</td>
</tr>
</tbody>
</table>

3.3 EC: The energy charge (EC) is the cost per kilowatt-hour for fuel and other expenses that are directly related to the production of electrical energy. Although the costs per kilowatt-hour will vary with the level of demand, a single energy charge representing an average cost per kilowatt-hour throughout the load cycle should be used for the sake of simplicity. Equations 1 and 2 are based on the assumption that the energy charge remains constant throughout the life of the transformer. See Section 4 for a discussion of the effects of inflation and increasing costs on the energy charge. If power is purchased, EC will be the kWh (or energy) cost of power.
3.4 **K**: The peak responsibility factor (K) is intended to compensate for the transformer peak load losses not occurring at the system peak losses. This means that only a fraction of the peak transformer losses will contribute to the system peak demand. The value of K can be determined from:

\[
\frac{\text{Peak responsibility factor (K)}}{\text{Transformer load at time of system peak}} = \frac{\text{Transformer peak load}}}{(Eq. 4)}
\]

It should be pointed out that K is squared in Equations 2 and 3 because K is a ratio of loads while losses are proportional to the load squared. Any value of K that seems appropriate can be used. The following are recommended values that appear to be reasonable.

<table>
<thead>
<tr>
<th>Transformer Type</th>
<th>K</th>
<th>K^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator step-up</td>
<td>1.0</td>
<td>1.00</td>
</tr>
<tr>
<td>Transmission substation</td>
<td>0.9</td>
<td>0.81</td>
</tr>
<tr>
<td>Distribution substation</td>
<td>0.8</td>
<td>0.64</td>
</tr>
</tbody>
</table>

3.5 **LFT**: The transformer loss factor is defined as the ratio of the average transformer losses to the peak transformer losses during a specific period of time. For the sake of simplicity, the equations assume that the transformer loss factor is a constant and that it does not change significantly over the life of the transformer.

The transformer loss factor can be determined directly using the equation:

\[
\text{Transformer loss factor (LFT)} = \frac{kW - \text{hours of loss during a specified time period}}{(\text{Hours}) (\text{Peak loss in kW in this period})} (Eq. 5)
\]

LFT can also be approximated from the load factor (the average load divided by the peak load for a specified time period) using the empirical equation below:

\[
\text{Transformer loss factor (LFT)} = 0.8 \times (\text{load factor})^2 + 0.2 \times (\text{load factor}) (Eq. 6)
\]

Where:

\[
\text{Load Factor} = \frac{\text{kWh per year}}{8760 \times \text{peak kW}}
\]
Load factor is the ratio of the average load over a period of time to the peak load occurring in that period. The load factor is a commonly available system parameter. The one-hour integrated peak value should be used.

3.5.1 Example: Determine the transformer loss factor for a substation transformer that has a load factor of 47 percent.

3.5.2 Solution:

Transformer loss factor = $0.8 \times (0.47)^2 + 0.2 \times (0.47)

Transformer loss factor = 0.271

3.6 \textbf{G}: The peak ratio is defined by the equation:

$$\text{Peak ratio}_G = \left( \frac{\text{Peak annual transformer load}}{\text{Full rated transformer load}} \right)^2 \quad (\text{Eq. 7})$$

For the peak annual transformer load, the one hour integrated peak value should be used.

The purpose of the peak ratio is to relate the value of Equation 2 to the full rated transformer load and not to the peak transformer load that would otherwise result if G were not in the equation.

If the total kVA of all transformers is known for your system and the peak kW (or kVA) load is known, then the average peak ratio for your system would be:

$$\text{Peak ratio}_G = \left( \frac{\text{Peak kVA load}}{\text{Total kVA of all transformers}} \right)^2$$

If the peak kW is known, but the peak kVA is unknown, assume a reasonable power factor on peak and calculate peak kVA as follows:

$$\text{kVA} = \frac{\text{kW}}{\text{power factor}}$$

If the transformer being purchased has a peak ratio different from the average, use that value. If the transformer will be installed at a known substation, use the billing data and assumed load growth for that substation.

The equations above are based on the assumption that the peak annual transformer load remains the same throughout the life of the transformer. If the load on the transformer is expected to increase annually, then use a reasonable equivalent level yearly peak load value based on
experience even though the expected peak loading value will increase every year. Another method is to calculate a value using Equation A-4 which is explained and derived in Appendix A. Equation A-4 yields an equivalent level load that will result in the same total losses as the actual non-level loading pattern.

3.7 LFA: The auxiliary loss factor compensates for the transformer auxiliary equipment that operates during only part of the transformer’s load cycle. For a transformer with two stages of cooling:

\[
LFA = (0.5) \cdot (\text{probability first stage of cooling will be on at any given time}) + (0.5) \cdot (\text{probability second stage of cooling will be on at any given time})
\]  

(Eq. 8)

The choice of the proper probabilities in the above equation is a matter of judgment based on historical system loading patterns. It is expected that the above probabilities under normal loading patterns will be extremely low. Since energy use and losses associated with transformer auxiliaries are extremely small over the life of the transformer, they could be ignored. The capital cost associated with auxiliaries are significant and should be considered.

4. INFLATION

The problem of dealing with inflation in economic studies is a difficult and complex topic that is frequently misunderstood. The purpose of this section is not to provide an in-depth analysis of the subject but rather to provide some general guidelines.

One method of handling inflation would be to increase future variable costs, such as the costs of losses, by the percentage represented by the general inflation rate. Since Equations 1, 2, and 3 do not have any provisions for costs that increase over the years, an equivalent level cost that takes into account future cost increases should be used. Equation 9 will yield such a value and can be used to adjust for inflation. (See Appendix B for derivation.)

\[
A' = A \cdot X \cdot \left[ \frac{1 - X^n}{1 - X} \right] \frac{i(1 + i)^n}{(1 + i)^n - 1}
\]

(Eq. 9)

where:

\[
X = \left( \frac{1 + r}{1 + i} \right) \text{ for } r \neq i
\]

(Eq. 10)

\[
A' = \text{the cost adjusted for inflation}
\]
A = the base cost before inflation

n = the number of years in the inflation period.
   It is recommended that “n” be taken as 35 years, which is the assumed transformer life.
   By assuming an “n” equal to the life of the transformer, an implicit assumption is being
   made that inflation will continue throughout the life of the transformer.

i = money interest rate per year expresses as a decimal, e.g., for 7%, i = .07)

r = the rate of inflation per year expressed as decimal; e.g., for 3% inflation, r = .03)

The term \[
\left( \frac{i}{1+i} \right)^n - 1
\] is called the capital recovery factor and tables for determining it are
easily available in most standard engineering economy texts and computer software. While the
above method increases future costs, it fails to take into account the value of money decreasing
with inflation.

Another method of handling inflation is to assume that the increase of costs in the future will be
balanced out by the decrease in the value of money, thus allowing us to ignore inflation
altogether. The problem with this approach is that the assumption does not always hold true
because costs of certain items may increase faster than the inflation rate.

A third method of treating inflation that appears more realistic than the two methods mentioned
previously is to compensate both for the increase in costs associated with the generation and
transmission of electric power and for the decrease in the value of the dollar due to the generally
prevailing inflation rate. This compensation can be accomplished by coming up with an
“equivalent inflation rate” (\(r'\)) that could be used in Equations 9 and 10. The formula for the
equivalent inflation rate follows:

\[
r' = \left[ \frac{1 + P}{1 + ig} \right] - 1
\]

for \(P \geq ig\)  
\[(Eq. 11)\]

where:

\(r'\) = the equivalent inflation rate

\(P\) = the rate of the increase in costs per kWh associated with power generation and
   transmission expressed as a decimal.

\(ig\) = the inflation rate for the economy as a whole expressed as a decimal.

The approximate form of the equation above is:

\[
r' = P - ig
\]

\[(Eq. 12)\]
4.1 **Example 1**: Find the equivalent inflation rate factor by which the energy charge rate should be adjusted to compensate for inflation if the following factors apply:

- General inflation rate \((ig) = 5\%\)
- Rate of increase of generating costs per kWh \((P) = 5\%\)
- Time value of money \((i) = 7\%\)

4.2 **Solution for Example 1**: 

Solving for the equivalent inflation rate:

\[
r' = \left[ \frac{1 + P}{1 + ig} \right] - 1
\]

\[
r' = \left[ \frac{1 + 0.05}{1 + 0.05} \right] - 1 = 0
\]

Solve Equation 10 for \(r' = 0\), we get:

\[
X = \left[ \frac{1 + 0}{1 + i} \right] = \left[ \frac{1}{1 + i} \right]
\]

Then solving Equation 9, we get:

\[
A' = A \cdot \left( \frac{1}{1 + i} \right) \left[ \frac{1 - \left( \frac{1}{1 + i} \right)}{1 - \left( \frac{1}{1 + i} \right)} \right]^{n} \left[ \frac{i(1 + i)^n}{(1 + i)^n - 1} \right]
\]

\[
A' = A \cdot \left( \frac{1}{1 + i} \right) \left[ \frac{(1 + i)[(1 + i)^n - 1]}{(1 + i)^n[(1 + i) - 1]} \right] \left[ \frac{i(1 + i)^n}{(1 + i)^n - 1} \right]
\]

\[
A' = A
\]

Thus, the cost adjusted for inflation is the same as the base cost. The above result was to be expected. Since the general inflation rate can be taken to be the rate at which the value of money decreases and since for this case it is equal to the rate of increase in costs, the two factors can be considered to cancel one another out and inflation can be ignored.

4.3 **Example 2**: Find the factor by which the energy charge rate should be adjusted to
compensate for inflation if the following factors apply:

- General inflation rate \( (ig) = 5\% \)
- Rate of increase of generating costs per kWh\( (P) = 6.5\% \)
- Time value of money \( (i) = 8\% \)

4.4 **Solution for Example 2:** Solving for the equivalent inflation rate:

\[
\text{Equation (exact)} \quad r' = \left[ \frac{1 + P}{1 + ig} \right] - 1 - \left[ \frac{1 + 0.065}{1 + 0.05} \right] - 1 = 0.0143
\]

Equation (approximate): \( r' = P - ig = 0.065 - 0.05 = 0.015 \)

Solving Equation 9 for \( r = 0.0143 \), and \( n = 35 \) years:

\[
x = \frac{1 + 0.0143}{1 + 0.08} = 0.9392
\]

From engineering economics tables, the capital recovery factor for \( i = 8\% \) and \( n = 35 \) years, is 0.08580. Therefore:

\[
A = A \cdot (0.9392) \left[ \frac{1 - (0.9392)^{35}}{1 - 0.9392} \right] (0.08580)
\]

\[
A' = A \cdot (1.178)
\]

In general, the key to properly accounting for inflation in economic studies is to realize that inflation increases not only dollar costs, but decreases the value of the dollar and affects all other factors that are related to money and the time value of money.

5. **EXAMPLE**

A 161/34.5 kV transformer rated at 30/40/50 MVA is to be purchased. This transformer is to be installed in a substation located at the end of an 80 mile (128 km) transmission line. Determine the load, no-load, and auxiliary loss evaluation values in dollars per kilowatt of the guaranteed losses at the 50 MVA rating.

Assume:

- Capital cost of power plant is $1,000/kW.
• Capital cost of line and associated facilities is $130/kW.
• Average energy cost is $0.02/kWh.
• Carrying charge rate is 14.6%.
• Time value of money is 9%.
• Load factor will stay at a constant value of 53% throughout the life of the transformer.
• Annual peak load will remain constant at a value of 53 MVA.
• Non-capital costs associated with generation and transmission increase at 5% per year.
• General inflation rate is 4%.

5.1 The first solution step is to adjust the energy charge for the difference between the general inflation rate and the inflation of costs.

Thus, the energy charge adjusted for inflation:

\[
EC' = EC \cdot X \cdot \left[ \frac{1 - X^n}{1 - X} \right] \left[ \frac{i (1 + i)^n}{(1 + i)^n - 1} \right] \\
X = \left[ \frac{1 + r}{1 + i} \right]
\]

Adjusted inflation rate:

\[
r' = P - ig = 5 - 4 = 1\%
\]

\[
X = \left[ \frac{1+.01}{1+.09} \right] = 0.927
\]

Assuming that inflation will continue into the unforeseeable future, a value of \(n=35\) will be used, as 35 years is the assumed life of a transformer.

\[
EC' = EC \cdot (0.927) \left[ \frac{1 - 0.927^{35}}{1 - 0.927} \right] \left[ \frac{0.09(1 + 0.09)^{35}}{(1 + 0.09)^{35} - 1} \right]
\]

\[
EC' = (0.027) (12.73) (0.0946)
\]

\[
EC' = $0.022 / kWh
\]
5.2 The system capital investment is equal to the cost of the plant plus the cost of transmission and associated facilities, all per kW; thus:

\[ \text{SI} = \$1,000 \text{ / kW} + \$130 \text{ / kW} = \$1,130 \text{ kW} \]

5.3 Solving Equation 1 for the cost of no-load losses in dollars per kilowatt of losses:

\[
\text{Cost of no-load losses in dollars per kW of loss} = \text{SI} + \frac{8760 \cdot EC'}{FCR} \\
= 130 + \frac{8760 \cdot 0.022}{0.146} \\
= \$2,450 / \text{kW of no-load loss}
\]

5.4 Solving Equation 2 for the cost of the load losses in dollars per kilowatt of losses:

\[
\text{Cost of load losses in dollars per kW of loss} = (\text{SI}(K^2)(G)) + \frac{8760 \cdot EC' \cdot (LFT)(G)}{FCR}
\]

According to Section 3.4, a peak responsibility factor (K) of 0.8 would be appropriate.

The peak ratio:

\[
G = \left[ \frac{\text{peak annual transformer load}}{\text{full rated transformer load}} \right]^2 \\
G = \left[ \frac{53 \text{ MVA}}{50 \text{ MVA}} \right]^2 = 1.124
\]
Transformer loss factor:

\[
LFT = 0.8(0.53)^2 + 0.2(0.53)
\]

\[
LFT = 0.331
\]

\[
\text{Cost of load losses in dollars per kW of loss} = 1130(0.8)^2 (1.124) + \frac{8760(0.022)(0.331)(1.124)}{0.146}
\]

\[
= \$1,304/\text{kW of load losses at 50 MVA}
\]

### 5.5 Solving Equation 3 for the cost of the auxiliary losses:

\[
\text{Cost of auxiliary losses per kW of loss} = S I(K^2) + \frac{8760 \bullet EC \bullet LFA}{FCR}
\]

From the system loading pattern, it is judged that the probability that the first stage of cooling will be on at any one time is 0.04 and that the probability that the second stage of cooling at any one time is 0.01. Thus, the loss factor for all auxiliary equipment operating is as follows:

\[
LFA = 0.5(0.04) + 0.5(0.01)
\]

\[
LFA = 0.025
\]

\[
\text{Cost of auxiliary losses all running} = 1130(0.8^2) + \frac{8760(0.022)(0.025)}{0.146}
\]

\[
= \$723 + 33.0
\]

\[
\text{Cost of auxiliary losses all running} = \$756
\]

The three loss values are:

<table>
<thead>
<tr>
<th>No-Load Core Losses</th>
<th>Load(Copper) Losses</th>
<th>Auxiliary Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2,450/kW</td>
<td>$1,304/kW</td>
<td>$756</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

This bulletin provides formulae, energy loss concerns, and examples of a suggested method for evaluating transformers for purchase. Borrowers and others could use the method presented as part of a standard procurement practice to ensure that the most economical, long-term, purchasing decision is achieved.

However, because of the many variables involved, such as inflation rates, peak loading times, investment costs, etc., users of this evaluation method should exercise judgment when using the formulae.
APPENDIX A: EQUIVALENT LEVEL LOAD FORMULA

For the equivalent level load formula, it is assumed that the load on the transformer is initially \( Y_1 \), load on the transformer increases at a rate of \( k \) percent per year, and when the load reaches \( Y_2 \), the transformer is either changed out or a second transformer is installed. It is further assumed that this pattern continues for the life of the transformer.

The equivalent level load is an equivalent level loss value that results in the same total losses as the yearly increasing load. This loss value can then be equated to a load.

Let:

- \( k \) = the rate of growth of the load, expressed as a decimal
- \( Y_1 \) = the ratio of load to capacity when transformer is installed
- \( Y_2 \) = the ratio of load to capacity when transformer is removed or supplemented
- \( t \) = time in years that it takes for load to grow from \( Y_1 \) to \( Y_2 \).

If it is assumed that the load growth cycle on the transformer is repeated throughout its life, then only the equivalent level load for one cycle is required.

Since losses are proportional to the square of the load:

\[
\text{LS} = \frac{a \int_0^t Y_1^2 (1 + K)^{2n} \, dn}{t} \quad \text{(Eq. A-1)}
\]

Where:

- \( \text{LS} \) = equivalent level losses
- \( n \) = time in years
- \( a \) = proportionality factor between losses and load such that:

\[
\text{LS} = a(\text{load})^2 \quad \text{(Eq. A-2)}
\]
Completing the integral and solving Equation A-1 results in the following for LS:

\[
LS = aY_1^2 \left( \frac{(1+k)^{2t} - 1}{2t \ln(1+k)} \right)
\]

(Eq. A-3)

Setting Equation A-2 equal to Equation A-3, we solve for “load”, the equivalent level peak load and end up with:

\[
\left[ \text{Equivalent level peak load} \right] = Y_1 \sqrt{\frac{(1 + k)^{2t} - 1}{\ln (1 + k)^{2t}}}
\]

(Eq. A-4)

The derivation of time “t” is as follows:

\[
Y_2 = Y_1 (1 + k)^t
\]

\[
\frac{Y_2}{Y_1} = (1 + k)^t
\]

\[
\log \frac{Y_2}{Y_1} = t \log (1 + k)
\]

\[
t = \frac{\log \left( \frac{Y_2}{Y_1} \right)}{\log (1 + k)}
\]

(Eq. A-5)

Where it cannot be assumed that the loading pattern on a transformer will be repeated throughout its life, the rationale used in the derivation above can still be used to determine the equivalent level loss value. The equivalent level loss value can be determined by dividing the integral of the square of the load curve by the time period involved.

Example

On a particular system a triple-rated oil-filled transformer is to be installed where the peak load is 0.95 of the self-cooled (OA) rating. The load is assumed to grow at 8 percent per year. When the peak load reaches 1.9 times the OA rating, an additional unit will be installed.
Assuming the above pattern will continue throughout the life of the transformer, determine the equivalent level peak load.

**Solution**

Substituting known values into Equation A-5, we find:

\[
\log \left( \frac{1.9}{0.95} \right) \\
\log(1 + 0.08) = 9.0 \text{ years}
\]

Then, substituting \( k = 0.08 \), \( t = 9 \) years, and \( Y_1 = 0.95 \) into Equation A-4, we find:

\[
[\text{Equivalent level peak load}] = 0.95 \times \sqrt{\frac{(1 + 0.08)^2(9.0) - 1}{(2)(9.0) \ln (1 + 0.08)}} = 1.40 \text{ of OA rating}
\]
APPENDIX B: EQUIVALENT LEVEL COST FORMULA

The following provides a derivation of Equation 9 which is used to adjust cost evaluations for inflation. Equation 9 will produce a level cost (or energy charge rate) that will yield the same total present worth value as a cost (or energy charge rate) that is increasing at “r” percent per year.

The present worth of a cost increasing at “r” percent per year is:

\[ PW = A \left[ \frac{(1+r)}{(1+i)} + \frac{(1+r)^2}{(1+i)^2} \ldots + \frac{(1+r)^n}{(1+i)^n} \right] \]  

(Eq. B-1)

where:

- \( i \) = the time value of money
- \( r \) = the rate of inflation
- \( A \) = the cost before inflation
- \( PW \) = the present worth
- \( n \) = the time period in years

If we let \( X = \frac{1+r}{1+i} \)  

(Eq. 10)

then:

\[ PW = A \cdot X \cdot (1 + X + X^2 \ldots X^{n-1}) \]

Algebraic manipulation yields the following:

\[ PW = AX \left[ \frac{1 - X^n}{1 - X} \right] \]  

(Eq. B-2)

Multiplying the above by the capital recovery factor to get an equivalent level yearly cost yields:

\[ A' = AX \left[ \frac{1 - X^n}{1 - X} \right] \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] \]  

(Eq. 9)